Series and parallel resistances

Objectives

- Calculate the equivalent resistance for resistors connected in both series and parallel combinations.
- Construct series and parallel circuits of lamps (resistors).
- Observe and explain relative lamp brightness in series and parallel circuits.

Assessment

1. Two resistors with resistances of 10 Ω and 30 Ω are connected in series with a 20 volt battery.
   a) What is the equivalent resistance of the circuit?
   b) What is the current flow through the circuit?

Assessment

2. Two resistors with resistances of 10 Ω and 30 Ω are connected in parallel with a 20 volt battery.
   a) What is the equivalent resistance of the circuit?
   b) What is the current flow through the circuit?

Assessment

3. If you connect these two identical resistors as shown, will they together draw more or less current than one of the resistors alone?

Physics terms

- series circuit
- parallel circuit
- equivalent resistance
Equations

Equivalent resistance for resistors connected in series

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

Equivalent resistance for resistors connected in parallel

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

Series circuits

A series circuit has only one path for the flow of electric current.

Parallel circuits

In a parallel circuit the electric current can split apart and come back together again.

Investigation

Part 1: Connecting lamps in series

1. Connect a +3 V voltage source (or two D-cell batteries) to the breadboard.
2. Insert the lamp into the breadboard to create a one-lamp circuit. Observe its brightness.

Predict: What will happen to lamp brightness if you connect a second lamp in series?

Part 1: Connecting lamps in series

3. Insert one lead of the same lamp/socket into one row of the breadboard...

\[ \text{...and the other lead of the same lamp/socket into a different row of the breadboard.} \]

Investigation

Part 1: Connecting lamps in series

3. Create a circuit with two lamps in series. Compare the brightness of the two lamps to the previous circuit with one lamp.

Was your prediction correct?
Investigation
Questions for Part 1
a. What property makes this a series circuit?
b. How bright are the lamps in series compared to the single lamp? Why?
c. Remove one lamp from the series circuit. What happens to the other lamp? Why?

Investigation
Questions for Part 2
a. What property makes this a parallel circuit?
b. How bright are the parallel lamps compared to the series lamps? Compared to the single lamp? Why?
c. Remove one lamp from the parallel circuit. What happens to the brightness of the other lamp? Why?

Investigation
Part 2: Connecting lamps in parallel
1. Create a circuit with two lamps in parallel.
2. Compare the brightness of the lamps in this circuit to the prior circuit with two lamps in series.

Which arrangement has more current flow through the lamps?

Designing a circuit
Design a circuit of three lamps that combines series and parallel arrangements.
• Sketch the circuit diagram.
• Predict the relative bulb brightness.
• Build the circuit and test your predictions.
• Is there a second way you could have designed your circuit?

Series circuits
The battery and the resistors in these circuits are identical.
What is different about these circuits?

Series circuits
The circuit with 3 resistors is 3 times harder for the current to flow through.
When you add resistors in series, equivalent resistance increases and current flow decreases: $I_1 < I_2 < I_3$
What is “equivalent resistance”?

- Equivalent resistance is the total combined resistance of a group of resistors.
- Equivalent resistance has the value of a single resistor that could replace the multiple resistors in a circuit, while keeping total current through the circuit the same.

Equivalent resistance: series

For resistors in series, you can find the equivalent resistance by simply adding up the individual resistances:

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

Engaging with the concepts

A 10 \( \Omega \) resistor, a 15 \( \Omega \) resistor, and a 5 \( \Omega \) resistor are connected in series. What is the equivalent resistance of this arrangement?

\[ 10 \Omega + 15 \Omega + 5 \Omega = 30 \Omega \]
Two strings of tree lights, each with a resistance of 150 Ω, are connected together in series. What is the \( R_{eq} \)?

Engaging with the concepts

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Engaging with the concepts

Two strings of tree lights, each with a resistance of 150 Ω, are connected together in series. What is the \( R_{eq} \)?

Engaging with the concepts

\[ R_{eq} = 300 \, \Omega \]

When you add resistors in series does \( R_{eq} \) increase or decrease? \( R_{eq} \) increases

Does total current increase or decrease? \( I \) decreases

What is the \( R_{eq} \) of each circuit?

\[ R_{eq} = 10 \, \Omega \]
\[ R_{eq} = 15 \, \Omega \]

How much current flows?
How much current flows?

\[ I = \frac{V}{R} = \frac{10 \text{ V}}{10 \Omega} = 1 \text{ A} \]
\[ I = \frac{V}{R} = \frac{10 \text{ V}}{15 \Omega} = 0.67 \text{ A} \]

Parallel circuits

A parallel circuit has more paths for electric current to flow. The circuit on the right lets twice as much current flow.

What is the \( R_{eq} \) for the parallel circuit?

\[ R_{eq} = \frac{1}{2} R \]

Parallel circuits

In the circuit with parallel resistors: total current flow doubles because total resistance is halved.

What is the \( R_{eq} \) for the parallel circuit?

\[ R_{eq} = \frac{1}{2} R \]

Equivalent resistance: parallel

When you add resistors in parallel, total resistance goes DOWN. To find the \( R_{eq} \) you must add the inverse of the resistances:

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

If you have two 4 \( \Omega \) resistors in parallel, what is the equivalent resistance?

A. 2 \( \Omega \)  
B. 4 \( \Omega \)  
C. 8 \( \Omega \)  
D. 1 \( \Omega \)

Don't forget to flip the fraction at the end!
A 10 Ω resistor and a 15 Ω resistor are connected in parallel. What is the $R_{eq}$ of this arrangement?

$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$

When resistors are connected in parallel, total resistance decreases.

Engaging with the concepts

Two strings of lights, each with a resistance of 150 Ω, are connected together. What is the $R_{eq}$ when the strings are connected in parallel?

$\frac{1}{R_{eq}} = \frac{1}{150} + \frac{1}{150} = \frac{1}{75}$

The total resistance is always less than any of the individual resistors added in parallel!
These are all 10 Ω resistors. What is the $R_{eq}$ in each case?

$$R_{eq} = 10 \, \Omega$$

$$R_{eq} = 5 \, \Omega$$

$$R_{eq} = 3.3 \, \Omega$$

Adding resistors in parallel makes the total resistance decrease.

Adding resistors in parallel makes the total resistance decrease.

How much current flows?

Apply Ohm's law ($V = IR$) to find the total current through the battery.

For the first circuit:

$$R_{eq} = 10 \, \Omega$$

$$I = \_\_\_$$

For the second circuit:

$$R_{eq} = 5 \, \Omega$$

$$I = \_\_\_$$

For the third circuit:

$$R_{eq} = 3.3 \, \Omega$$

$$I = \_\_\_$$

How much current flows?

Apply Ohm's law ($V = IR$) to find the total current through the battery.

For the first circuit:

$$R_{eq} = 10 \, \Omega$$

$$I = 3 \, A$$

For the second circuit:

$$R_{eq} = 5 \, \Omega$$

$$I = 6 \, A$$

For the third circuit:

$$R_{eq} = 3.3 \, \Omega$$

$$I = 9 \, A$$

Analogy for resistors in series

What happens to the potential energy of the water as it passes through each water wheel?

How is this analogous to the voltage drops across each resistor?

Voltage drops: resistors in series

Half as much current flows in the series circuit.

What are the voltage drops across the two resistors in series?

Voltage drops: resistors in series

Voltage drops: resistors in series

$$V = IR$$

Single resistor

Resistor in series
Voltage drops: resistors in parallel

Twice as much current flows in the parallel circuit.

What are the voltage drops across the resistors in parallel?

Voltage drops: resistors in parallel

Single resistor

$V = IR = (4A)(5\Omega) = 20$ V drop

Each resistor in parallel has the full 20 V drop.

Assessment

1. Two resistors with resistances of 10 Ω and 30 Ω are connected in series with a 20 volt battery.
   a) What is the equivalent resistance of the circuit?
   $R_{eq} = R_1 + R_2 = 10 \Omega + 30 \Omega = 40 \Omega$
   b) What is the current flow through the circuit?
   $I = \frac{V}{R} = \frac{20 V}{40 \Omega} = 0.50$ amps

Assessment

1. Two resistors with resistances of 10 Ω and 30 Ω are connected in parallel with a 20 volt battery.
   a) What is the equivalent resistance of the circuit?
   $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{10 \Omega} + \frac{1}{30 \Omega}} = \frac{1}{0.1 \Omega + 0.033 \Omega} = \frac{1}{0.133 \Omega} = 7.5 \Omega$
   b) What is the current flow through the circuit?
   $I = \frac{V}{R} = \frac{20 V}{7.5 \Omega} = 2.67$ amps
2. Two resistors with resistances of 10 Ω and 30 Ω are connected in parallel with a 20 volt battery.

   a) What is the equivalent resistance of the circuit? 7.5 Ω
   \[
   \frac{1}{R_{eq}} = \frac{1}{10 \, \Omega} + \frac{1}{30 \, \Omega} = \frac{4}{30 \, \Omega} \quad \text{so} \quad R_{eq} = \frac{30 \, \Omega}{4} = 7.5 \, \Omega
   \]
   b) What is the current flow through the circuit?

   \[I = \frac{V}{R_{eq}} = \frac{20 \, \text{V}}{7.5 \, \Omega} = 2.7 \, \text{amps}\]

Assessment

3. If you connect these two identical resistors as shown, will they together draw more or less current than one of the resistors alone?

   They will draw more current, because they are connected in parallel. There are now two different, but identical paths for electricity to flow, so the current doubles!

Assessment: lamps in series

How bright are the lamps in series compared to the single lamp? Why?

They are dimmer. The total resistance increases, so the current decreases.
When you remove one of the lamps in series, what happens to the brightness of the other lamp? Why?

It goes out. Removing the bulb results in an open circuit. No current flows.

Assessment: lamps in series

When you remove one of the lamps in parallel, what happens to the brightness of the other lamp? Why?

The other lamp remains on, and its brightness does not change. The voltage and resistance of the lamp is unchanged so the current through it is unchanged.

Assessment: lamps in parallel

How bright are the lamps in parallel compared to the series lamps? Why?

The parallel bulbs are brighter. They each get twice as much current as a series bulb.

Assessment: lamps in parallel

How bright are the lamps in parallel compared to the series lamps? Why?

The parallel bulbs are brighter. They each get twice as much current as a series bulb.

Assessment: lamps in parallel